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A NONLINEAR HYPERBOLIC VOLTERRA EQUATION ARISING IN HEAT FLOW.(U)
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A NONLINEAR HYPERBOLIC VOLTERRA EQUATION ARISING IN HEAT FLOW

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ABSTRACT

A mathematical model for nonlinear heat flow in a rigid body of material with memory leads to the integrodifferential equation problem:

$$
\text{(HF)} \quad \begin{cases} u_t(t,x) = \int_0^t a(t - \tau)\sigma(u_x(t,x))_x d\tau + f(t,x) & (0 < t < \infty, \quad x \in \mathbb{R}) \\ u(0,x) = u_0(x) & (x \in \mathbb{R}) , \end{cases}
$$

which is analyzed by an energy method developed jointly with C. M. Dafermos. Global existence, uniqueness, boundedness and the decay of smooth solutions as $t \to \infty$ are established for sufficiently smooth and "small" data, under physically reasonable assumptions.

AMS (MOS) Subject Classifications: 45K05, 47H15, 47H10, 45M99, 35L60

Key Words: Nonlinear Volterra integrodifferential equations (of hyperbolic type), Global existence and uniqueness, Smooth solutions, Boundedness, Asymptotic behaviour, Energy method, Resolvent kernels, Frequency domain method, Nonlinear heat flow in materials with memory

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SIGNIFICANCE AND EXPLANATION

Problem (HF) stated in the Abstract is a mathematical model for nonlinear heat flow (with a finite speed of propagation) in an unbounded one-dimensional rigid body of material with "memory". With an appropriate interpretation of the term $\sigma(u_x)_x$ (HF) has a valid physical meaning in two and three space dimensions.

To motivate the rather technical assumptions concerning the kernel a used in the analysis we derive (HF) from physical principles using the internal energy and the heat flux in such a body, expressed as certain functionals of the temperature and of the gradient of the temperature respectively, and then applying the balance of heat law. The assumptions on a are then motivated by physical and partly mathematical arguments involving the internal energy and heat flux response functions. Such arguments seem to have been omitted in earlier literature.

Problem (HF) cannot conveniently be solved explicitly, even in the linear case ($\sigma(\xi) = c^2\xi$, $\xi \in \mathbb{R}$, c a constant). We use an energy method developed jointly with C. M. Dafermos (MRC Technical Summary Report #1876 - Communications in PDE, to appear) to explain that, under physically reasonable assumptions on the functions a, σ, f, u_0, problem (HF) has a unique, global smooth solution $u \in C^2([0,\infty) \times \mathbb{R})$, provided the data u_0 and f are sufficiently smooth and "small". Moreover, this solution has a finite speed of propagation and possesses certain boundedness and decay properties as $t \to \infty$. The restriction to small data is needed to preclude the development of "shock" solutions due to possible crossing of characteristics of a related nonlinear wave equation. An interpretation of the result is that the memory term in (HF) provides a dissipative mechanism which has a smoothing effect for "small", smooth data, under physically reasonable conditions on the kernel a.

The energy method rests on an auxiliary inequality (equation (3.8)) which is derived by a simple "frequency domain" argument using the properties of a. The remarks following Lemma 3.3 suggest new and physically meaningful conditions on a (expressible in terms of the energy and heat flux response functions) for the application of the energy method. Such generalizations are under active study.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

A NONLINEAR HYPERBOLIC VOLTERRA EQUATION ARISING IN HEAT FLOW

John A. Nohel

Introduction. In this largely expository paper which is based on recent joint work with C. M. Dafermos [1] we use energy methods to discuss the global existence, uniqueness, boundedness, and decay as $t \to \infty$ of smooth (C^2) solutions of the nonlinear Cauchy problem

$$(\mathrm{HF}) \begin{cases} u_t(t,x) = \int_0^t a(t-s)\sigma(u_x(s,x))_x ds + f(t,x) & (0 < t < \infty,\ x \in \mathbb{R}) \\ u(0,x) = u_0(x) & (x \in \mathbb{R}) \end{cases}$$

for appropriately smooth and "small" data u_0, f. Here $a : [0,\infty) \to \mathbb{R}$, $\sigma : \mathbb{R} \to \mathbb{R}$ ($\sigma(0) = 0$), $f : [0,\infty) \times \mathbb{R} \to \mathbb{R}$, $u_0 : \mathbb{R} \to \mathbb{R}$ are given functions satisfying assumptions motivated partly by physical considerations sketched below, and partly by the method of analysis; subscripts in (HF) denote partial derivatives. Some comments on closely related initial-boundary value problems are made following the statement of the main result (Theorem 2.1). With appropriate interpretation of the term $\sigma(u_x)_x$, problem (HF) has a valid physical meaning in any number of space dimensions, and we refer to [1, Thm. 7.1] to such a problem in $\mathbb{R}^2$ studied by an extension of this method. An earlier study of (HF) by Mac Camy [6] is based on the method of Riemann invariants and is therefore restricted to a single space dimension. The present method which yields more widely applicable results even

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in one space dimension is more direct and simpler. For a similar approach to a problem in nonlinear viscoelasticity we refer the reader to [1, Theorem 5.1] and to Mac Camy [7] for the Riemann invariant approach.

To motivate the assumptions to be imposed, particularly with regard to the kernel a, we consider briefly the problem of nonlinear heat flow in an unbounded one-dimensional rigid body of a material with memory. Let $u(t,x)$, $\varepsilon(t,x)$, $q(t,x)$, and $h(t,x)$ denote respectively the temperature, the internal energy, the heat flux, and the external heat supply at time t and position x. Following Gurtin and Pipkin [2], and also Mac Camy [6], we assume a model for heat flow in which $\varepsilon(t,x)$ and $q(t,x)$ are respectively the following functionals of the temperature u and of the gradient of temperature u_x:

$$(1.1)\qquad \varepsilon(t,x) = bu(t,x) + \int_{-\infty}^{t} \beta(t-s)u(s,x)ds \qquad (0 < t < \infty,\ x \in \mathbb{R});$$

$$(1.2)\qquad q(t,x) = -\int_{-\infty}^{t} \gamma(t-s)\sigma(u_x(s,x))ds \qquad (0 < t < \infty,\ x \in \mathbb{R});$$

it is assumed that the history $u_0(t,x)$ of u (and hence also the history $u_{0x}(t,x)$ of the temperature gradient) is prescribed up to $t = 0$ and for $x \in \mathbb{R}$. We can assume without loss of generality that $u_0(t,x) \equiv 0$, $(t < 0$, $x \in \mathbb{R})$; for if that is not the case, it is easily seen from what follows that this merely alters the forcing term f in (HF). It is reasonable to assume that $\sigma : \mathbb{R} \to \mathbb{R}$ is a smooth function satisfying $\sigma(0) = 0$, $\sigma'(0) > 0$ (in fact, $\sigma'(\xi) \geq \varepsilon > 0$, $(\xi \in \mathbb{R})$ - recall that for linear heat flow $\sigma(\xi) = c\xi$, $c > 0$ a constant). We shall assume that $b > 0$ is a given constant and that the given smooth "memory" functions $\beta, \gamma \in L^1(0,\infty)$; thus $\varepsilon(t,x)$ and $q(t,x)$ are bounded whenever $u(t,x)$ and $u_x(t,x)$ are bounded. It should be noted that in the applied literature β, γ are linear combinations of decaying exponentials with positive coefficients.

If $h(t,x)$ denotes the external heat supply, the balance of heat requires that

$$(1.3)\qquad \varepsilon_t(t,x) = -q_x(t,x) + h(t,x) \qquad (0 < t < \infty,\ x \in \mathbb{R}).$$

Substituting (1.1), (1.2) into (1.3), and using the assumption that $u(t,x) \equiv 0$ for $t < 0$, $x \in \mathbb{R}$ yields

$$bu_t(t,x) + \frac{\partial}{\partial t}\int_0^t \beta(s)u(t-s,x)ds = \int_0^t \gamma(t-s)\sigma(u_x(s,x))_x ds + h(t,x) ,$$

or equivalently

$$(1.4) \qquad bu_t(t,x) + \int_0^t \beta(t-s)u_t(t-s,x)ds = \int_0^t \gamma(t-s)\sigma(u_x(s,x))ds$$
$$+ h(t,x) - \beta(t)u(0,x) ,$$

where we also prescribe the value $u(0,x) = u_0(x)$, $x \in \mathbb{R}$. To reduce (1.4) to (HF) define the resolvent kernel ρ of β by the relation

$$(\rho) \qquad b\rho(t) + (\beta*\rho)(t) = -\frac{\beta(t)}{b} \qquad (0 \leq t < \infty) ,$$

where here and in what follows $*$ denotes the convolution on $[0,t]$. It follows from standard theory of linear Volterra equations that (ρ) has a unique solution $\rho \in C^1[0,\infty)$. If g is a given function on $[0,\infty)$, the solution of the Volterra equation

$$(V) \qquad by(t) + (\beta*y)(t) = g(t) \qquad (0 \leq t < \infty)$$

is given by the variation of constants formula

$$(1.5) \qquad y(t) = \frac{g(t)}{b} + (\rho*g)(t) \qquad (0 \leq t < \infty) .$$

Applying (1.5) with $y = u_t$ and g the right-hand side of (1.4), one sees that (1.4) is equivalent to (HF) with

$$(1.6) \qquad a(t) = \frac{1}{b}\gamma(t) + (\rho*\gamma)(t) \qquad 0 \leq t < \infty$$

$$(1.7) \qquad f(t,x) = \frac{1}{b}(h(t,x) - \beta(t)u_0(x)) + \rho*(h(t,x) - \beta(t)u_0(x))$$
$$(0 \leq t < \infty,\ x \in \mathbb{R}) .$$

To motivate the assumptions to be imposed on a, we note from (1.1) and (1.5) that

$$u(t,x) = \frac{\varepsilon(t,x)}{b} + (\rho*\varepsilon)(t,x) \qquad (0 \leq t < \infty,\ x \in \mathbb{R}) ,$$

where ρ is the resolvent of β. If $\rho \in L^1(0,\infty)$ then ε bounded implies u bounded. But since we assumed $\beta \in L^1(0,\infty)$, the Paley-Wiener theorem applied to equation (ρ) yields that $\rho \in L^1(0,\infty)$ if and only if

$$\hat{\beta}(s) \neq -b \quad \text{for} \quad \operatorname{Re} s \geq 0$$

where $\hat{\beta}(s) = \int_0^\infty \exp(-st)\beta(t)dt$. But the internal energy ε is positive and so from (1.1)

$$b + \int_0^\infty \beta(t)dt = b + \hat{\beta}(0) > 0 \ .$$

Therefore, the Paley-Wiener condition can be modified to the statement: $\rho \in L^1(0,\infty)$ if and only if

$$\operatorname{Re} \hat{\beta}(s) + b > 0, \qquad (\operatorname{Re} s \geq 0) \ . \tag{1.8}$$

The assumption $\gamma \in L^1(0,\infty) \cap C^1[0,\infty)$, together with (1.6), then imply that $a \in L^1(0,\infty) \cap C^1[0,\infty)$, and an easy calculation using (ρ) and (1.6) shows that

$$\hat{a}(s) = \frac{\hat{\gamma}(s)}{b + \hat{\beta}(s)} \qquad (\operatorname{Re} s \geq 0) \ . \tag{1.9}$$

For physical reasons and (1.2) one needs to require $\int_0^\infty \gamma(t)dt > 0$, and so it also follows from (1.9) that

$$\int_0^\infty a(t)dt = \hat{a}(0) = \frac{\int_0^\infty \gamma(t)dt}{b + \int_0^\infty \beta(t)dt} > 0 \ . \tag{1.10}$$

Physically the function γ represents the heat flux relaxation function, and it is reasonable to assume that $\gamma(0) > 0$. It then follows from (1.6) that $a(0) = \frac{1}{b}\gamma(0) > 0$. If, as is reasonable, it is also assumed that $\gamma'(0) \leq 0$ and $\beta(0) > 0$, one also has from (1.6) that $a'(0) = \frac{1}{b}\gamma'(0) + \rho(0)\gamma(0) = \frac{1}{b}\gamma'(0) - \frac{\beta(0)\gamma(0)}{b} < 0$.

To summarize, the following assumptions concerning the kernel a in (HF) are reasonable for the heat flow problem:

$$a \in C^1[0,\infty) \cap L^1(0,\infty),\ a(0) > 0,\ a'(0) < 0,\ \int_0^\infty a(t)dt > 0 \ ; \tag{1.11}$$

as we shall see below we shall require additional smoothness of a, as well as a positivity "frequency domain" condition involving the Laplace transform of a. The implications of this condition are discussed in Lemma 3.3 and Remarks following it.

In the analysis of (HF) which follows we shall impose other technical assumptions (see assumptions (a), (σ), (f), (u_0) below). To motivate our result for (HF) assume for the moment that $a(t) \equiv a(0) > 0$ for $t \geq 0$. Then (HF) is formally equivalent to the Cauchy problem

$$\text{(W)} \qquad u_{tt} = a(0)\sigma(u_x)_x + f_t,\ u(0,x) = u_0(x),\ u_t(0,x) = u_1(x) = f(0,x)\ .$$

If σ is "genuinely nonlinear" ($\sigma''(\xi) \neq 0$, $\xi \in \mathbb{R}$), Lax [3] has shown that (W) fails to have global smooth solutions in time (even if $f_t \equiv 0$), no matter how smooth one takes the initial data due to the development of "shocks" (the first derivatives of u develop singularities in finite time due to the crossing of characteristics). Note that $a(t) \equiv a(0)$ is excluded by (1.11).

Nishida [8] has shown that for the nonlinear wave equation with frictional damping

$$(\text{W}_\alpha) \qquad u_{tt} + \alpha u_t = a(0)\sigma(u_x)_x,\ u(0,x) = u_0(x),\ u_t(0,x) = u_1(x),\quad a(0) > 0\ ,$$

the dissipation term αu_t, $\alpha > 0$, precludes the development of shocks if the initial data are sufficiently smooth and "small". The proof rests on the concept of Riemann invariant and is restricted to one space dimension. For a generalization of Nishida's method to the forced equation (W_α) we refer to Nohel [9]. As will be seen in (3.3) below, (HF), under physically reasonable assumptions, is equivalent to a variant of (W_α) with an additional memory term which makes our result for (HF) (Theorem 2.1) plausible. The Nishida approach applied to (HF) (necessarily restricted to one space dimension) was studied by Mac Camy [6].

2. Statement of Results.

We make the following assumptions. Concerning σ let

$$(\sigma) \qquad \sigma \in C^3(\mathbb{R}),\ \sigma(0) = 0,\ \sigma'(0) > 0\ ,$$

the first for technical reasons, the others on physical grounds (recall that in the linear version of (HF) $\sigma(u_x) = u_x$). Concerning the kernel a assume

$$
\text{(a)} \quad \begin{cases}
\text{(i)} & a \in B^{(3)}[0,\infty) \,, \\
\text{(ii)} & a(0) > 0, \; a'(0) < 0 \,, \\
\text{(iii)} & t^j a^{(m)} \in L^1(0,\infty) \qquad (j,m = 0,1,2,3) \,, \\
\text{(iv)} & \text{letting} \quad \hat{a}(i\eta) = \int_0^\infty e^{-i\eta t} a(t)\,dt \,, \\
 & \text{(2.1)} \quad \operatorname{Re} \hat{a}(i\eta) > 0 \qquad (\eta \in \mathbb{R}) \,,
\end{cases}
$$

where $B^{(m)}[0,\infty)$ is the set of functions with bounded continuous derivatives up to and including order m. From (1.11) above the conditions $a \in C^1(0,\infty)$, $a \in L^1(0,\infty)$, $a(0) > 0$, $a'(0) < 0$, $\hat{a}(0) = \int_0^\infty a(t)dt > 0$ are reasonable on physical grounds; the remaining ones are needed for technical reasons of the analysis. See additional remarks on alternatives to the frequency domain condition (2.1) following Lemma 3.3 below. Concerning the forcing term f we assume (essentially for technical reasons)

$$
\text{(f)} \qquad f, f_t, f_x, f_{tt}, f_{tx}, f_{xx}, f_{ttt}, f_{ttx}, f_{txx} \in L^2(0,\infty;L^2(\mathbb{R})) \,.
$$

The initial datum u_0 is assumed to satisfy

$$
(u_0) \qquad u_{0x}, u_{0xx}, u_{0xxx} \in L^2(\mathbb{R}) \,.
$$

Note that in (u_0) no assumption is explicitly made about $u_0(x)$; however, for the particular physical problem one would also have to require $u_0 \in L^2(\mathbb{R})$ in order that f defined by (1.7) satisfy (f). Our result concerning (HF) is (see [1; Theorem 4.1]):

Theorem 2.1. Let the assumptions (σ), (a), (f), (u_0) hold. If the $H^2(\mathbb{R})$ norm of u_{0x} and the $L^2([0,\infty);L^2(\mathbb{R}))$ norms of f and its derivatives listed in (f) are sufficiently small, then the Cauchy problem (HF) has a unique solution $u \in C^2([0,\infty) \times \mathbb{R})$ with the following properties:

(i) $u_t, u_x, u_{tt}, u_{tx}, u_{xx}, u_{ttt}, u_{ttx}, u_{txx}, u_{xxx} \in L^\infty([0,\infty);L^2(\mathbb{R}))$,

(ii) $u_t, u_{tt}, u_{tx}, u_{xx}, u_{ttt}, u_{ttx}, u_{txx}, u_{xxx} \in L^2([0,\infty);L^2(\mathbb{R}))$,

(iii) $u_t(t,\cdot), u_{tt}(t,\cdot), u_{tx}(t,\cdot), u_{xx}(t,\cdot) \to 0$ in $L^2(\mathbb{R})$ as $t \to \infty$,

(iv) $u_t(t,x), u_x(t,x), u_{tt}(t,x), u_{tx}(t,x), u_{xx}(t,x) \to 0$ as $t \to \infty$ uniformly on $\mathbb{R}$.

We remark that conclusions (iii), (iv) are easy consequences of (i), (ii). It also follows from the proof that the solution u has a finite speed of propagation.

We also note that the results of Theorem 2.1 hold (with essentially the same proofs) for the following two problems of heat flow in a body on the interval $[0,1]$ (see [1, Theorem 6.1]):

(i) (HF) on $(0,\infty) \times (0,1)$ with homogeneous Neumann boundary conditions at $x = 0$ and $x = 1$, and with $u_0(x)$ prescribed on $[0,1]$;

(ii) (HF) on $(0,\infty) \times (0,1)$ with homogeneous Dirichlet boundary conditions at $x = 0$ and $x = 1$, and with $u_0(x)$ prescribed on $[0,1]$.

In both problems assumptions (σ) and (a) are unchanged while assumptions (f) and (u_0) hold in $L^2[(0,1) : L^2(\mathbb{R})]$ and in $L^2(0,1)$ respectively. For problem (ii) one adds the assumption $f(t,0) = f(t,1) = 0$.

For a version of (HF) in two space dimensions and with a similar but technically more involved proof we refer the reader to [1, Theorem 7.1].

We note also that (HF) is of the abstract form

$$\text{(A)} \qquad \begin{cases} u'(t) + \int_0^t a(t-\tau)Au(\tau)d\tau = f(t) & (0 < t < \infty) \\ u(0) = u_0 , & \end{cases}$$

where A is the nonlinear operator $Au = -\frac{\partial}{\partial x}\sigma(u_x)$ plus appropriate conditions at $\pm\infty$ or suitable boundary conditions at $x = 0$ and 1. Such abstract problems have been recently studied by Londen [4], [5] for a class of kernels $a(\cdot)$ which are positive, decreasing, convex on $[0,\infty)$ and which satisfy the condition $a'(0+) = -\infty$ which is crucial for his technique. In addition, the solution obtained by Londen is not sufficiently regular, and no comparable decay results are obtained.

Finally, we observe that a comparison of Theorem 2.1 and of its proof with the results and method of proof by Mac Camy [6] shows that our approach is more direct, not restricted to one space dimension, and yields more general results (see additional remarks following Lemma 3.3).

3. Outline of Proof of Theorem 2.1. To simplify the exposition we shall assume that $f \equiv 0$ in (HF), and we refer the reader to [1] for the technically more involved treatment resulting from $f \not\equiv 0$; the method is unaltered by this simplification.

a. Transformation of (HF). Differentiation of (HF) with respect to t brings it to the form

$$(3.1)\quad \begin{cases} u_{tt}(t,x) = a(0)\sigma(u_x)_x(t,x) + (a'*\sigma(u_x)_x)(t,x) \\ u(0,x) = u_0(x),\ u_t(0,x) \equiv 0 \qquad (x \in \mathbb{R}) . \end{cases}$$

We transform (3.1) to an equivalent form by observing that this equation is linear in $y = \sigma(u_x)_x$. Define the resolvent kernel k of a' by the equation

$$(k)\qquad a(0)k(t) + (a'*k)(t) = -\frac{a'(t)}{a(0)} \qquad (0 \leq t < \infty) ;$$

since $a(0) > 0$, assumptions a(i) imply that k is uniquely defined and $k \in C^2[0,\infty)$ (k has other properties - see Lemma 3.3 below). By the variation of constants formula for linear Volterra equations one has

$$a(0)y + a'*y = \varphi \iff y = \varphi/a(0) + k*\varphi$$

for any given function φ. Applying this to (3.1) one sees that if u is a classical solution of (HF) with $f \equiv 0$, then u satisfies the equation

$$u_{tt} + a(0)k*u_{tt} = a(0)\sigma(u_x)_x .$$

Performing an integration by parts and using $u_t(0,x) \equiv 0$ shows that (HF) with $f \equiv 0$ is equivalent to the Cauchy problem

$$(3.2)\quad \begin{cases} u_{tt}(t,x) + a(0)\dfrac{\partial}{\partial t}(k*u_t)(t,x) = a(0)\sigma(u_x(t,x))_x \qquad (0 < t < \infty,\ x \in \mathbb{R}) \\ u(0,x) = u_0(x),\ u_t(0,x) \equiv 0 \qquad (x \in \mathbb{R}) . \end{cases}$$

Another important equivalent form of (HF) with $f \equiv 0$ resulting from (3.2) is

$$(3.3)\quad \begin{cases} u_{tt}(t,x) + a(0)k(0)u_t(t,x) = a(0)\sigma(u_x(t,x))_x - a(0)(k'*u_t)(t,x) \\ \qquad\qquad (0 < t < \infty,\ x \in \mathbb{R}) , \\ u(0,x) = u_0(x),\ u_t(0,x) \equiv 0 \qquad (x \in \mathbb{R}) . \end{cases}$$

Since $a(0)k(0) = -a'(0) > 0$, (3.3) suggests the dissipative mechanism induced by the memory term in (HF) and the relationship with the damped wave equation (W_α). We remark that if $f \not\equiv 0$ in (HF) one adds the forcing term $\phi(t,x) = f_t(t,x)/a(0) + (k*f_t)(t,x) + a(0)k(t)f(0,x)$ to (3.2) and (3.3), and one replaces the zero initial condition by $u_t(0,x) = f(0,x)$.

The proof of Theorem 2.1 is carried out in two stages: (i) A suitable local existence and uniqueness result is established. (ii) A priori estimates are established to continue the local solution; these will at the same time yield conclusions (i), (ii) of Theorem 2.1.

b. Local Theory. We shall make the temporary additional assumption concerning σ:

(σ^*) there exists $p_0 > 0$ such that $\sigma'(\xi) \geq p_0 > 0 \quad (\xi \in \mathbb{R})$.

Proposition 3.1. Let the assumptions (σ), (σ^*), (u_0) hold, and let $k', k'' \in C[0,\infty) \cap L^1(0,\infty)$. Then the Cauchy problem (3.2) (resp. (3.3)) has a unique solution $u \in C^2([0,T_0) \times \mathbb{R})$ on a maximal interval $[0,T_0) \times \mathbb{R}$, $T_0 \leq +\infty$, such that for $T \in [0,T_0)$ one has

(i) all derivatives of u of orders one to three inclusive $\in L^\infty([0,T]; L^2(\mathbb{R}))$;

(ii) if $T_0 < \infty$, then

$$\int_{-\infty}^{\infty} [u_t^2(t,x) + u_x^2(t,x) + u_{tt}^2(t,x) + \cdots + u_{xxx}^2(t,x)]dx \to \infty \quad \text{as} \quad t \to T_0^- .$$

We remark that the property of finite speed of propagation of solutions of (HF) is an easy consequence of the proof of Proposition 3.1.

The proof uses the Banach fixed point theorem. Let $X(M,T)$ be the set of functions $u \in C^2([0,T] \times \mathbb{R})$ for any $T > 0$ such that $u(0,x) = u_0(x)$, $u_t(0,x) = 0$ and such that

(i) $u_t, u_x, u_{tt}, \ldots, u_{xxx} \in L^\infty([0,T]; L^2(\mathbb{R}))$ and

(ii) $$\sup_{[0,T]} \int_{-\infty}^{\infty} [u_t^2(t,x) + u_x^2(t,x) + u_{tt}^2(t,x) + \cdots + u_{xxx}^2(t,x)]dx \leq M^2 .$$

Note that $X(M,T)$ is not empty if M is sufficiently large, and that if $u \in X(M,T)$, then

(iii) $$\sup_{[0,T]\times\mathbb{R}} \{|u_t(t,x)|, |u_x(t,x)|, |u_{tt}(t,x)|, |u_{tx}(t,x)|, |u_{xx}(t,x)|\} \leq M .$$

Let S be the map: $X(M,T) \to C^2([0,T] \times \mathbb{R})$ which carries a function $v \in X(M,T)$ into the solution of the linear Cauchy problem (see (3.3) for motivation)

$$
(3.4) \quad \begin{cases} u_{tt}(t,x) + k(0)u_t(t,x) = a(0)[\sigma'(v_x(t,x))u_{xx}(t,x) - (k'*v_t)(t,x)] \\ \qquad\qquad\qquad\qquad\qquad\qquad (0 < t < T,\ x \in \mathbb{R}) \\ u(0,x) = u_0(x),\ u_t(0,x) = 0\ . \end{cases}
$$

Clearly a fixed point of S will be a solution of (3.2) (respectively (3.3)). To apply the Banach fixed point theorem to the map S one first shows (by an energy argument, for details see [1, Lemma 3.1]) that if M is sufficiently large and if T is sufficiently small, then S maps $X(M,T)$ into itself. One next equips $X(M,T)$ with the metric

$$
\rho(u,\bar{u}) = \max_{[0,T]} \{\int_{-\infty}^{\infty} [(u_t(t,x) - \bar{u}_t(t,x))^2 + (u_x(t,x) - \bar{u}_x(t,x))^2]dx\}^{\frac{1}{2}}\ .
$$

By the lower semicontinuity of norms under weak convergence in Banach space, $X(M,T)$ becomes a complete metric space. One then shows that for M sufficiently large and T sufficiently small the map S is a strict contraction of $X(M,T)$ and the proof of Proposition 3.1 is completed in a standard manner (for details see [1, Lemma 3.2]).

If σ, k, u_0, u_1 are smoother, the solution becomes smoother. A precise regularity result which is needed for the a priori estimates is

Proposition 3.2. Let the assumptions of Proposition 3.1 be satisfied. In addition, assume that

$$
(3.5) \qquad \sigma \in C^4(\mathbb{R}),\ u_{0xxxx} \in L^2(\mathbb{R})\ .
$$

Then the solution u of Proposition 3.1 has the addition property

$$
(3.6) \qquad u_{tttt}, u_{tttx}, u_{ttxx}, u_{txxx}, u_{xxxx} \in L^{\infty}([0,T]; L^2(\mathbb{R}))\ ,
$$

for every $T < T_0$, where $[0,T_0) \times \mathbb{R}$ is the maximal interval of existence.

For the proof see [1, Theorem 3.2].

c. A Priori Estimates and Continuation. We wish to show that the maximal interval $[0,T_0)$ of Proposition 3.1 is in fact $[0,\infty)$. Recall that the local theory assumes that (σ^*) is satisfied; this assumption will be removed. The a priori estimates will be deduced from equation (3.2) above. We shall restrict the range of $u_x(t,x)$ for a local solution u to the set on which $\sigma'(\cdot) > 0$; choose $c_0 > 0$ such that

(3.7) $$\sigma'(w) \geq p_0 > 0, \; w \in [-c_0,c_0] .$$

We wish to show that there exists a constant $\mu > 0$, $\mu < c_0$, depending on p_0, $\int_0^\infty |k'(t)|dt$, $\max_{[-c_0,c_0]} \{|\sigma'(\cdot)|, |\sigma''(\cdot)|, |\sigma'''(\cdot)|\}$, but not on $T > 0$ such that if the local solution u of (3.1) satisfies

$$(\mu^*) \qquad \sup_{0\leq t<T, x\in\mathbb{R}} \{|u_t(t,x)|,|u_x(t,x)|,|u_{tx}(t,x)|,|u_{xx}(t,x)|\} \leq \mu ,$$

then certain functionals of the solution u are controllably small (i.e. these functionals can be made arbitrarily small by choosing the initial data sufficiently small in the appropriate H norms). More precisely, the result of the a priori estimates which follow is that if the assumptions of Theorem 2.1 hold (with $f \equiv 0$) and if the $H^2(\mathbb{R})$ norm of u_{0x} is sufficiently small, then for as long as the local solution u of (3.2) satisfies the condition (μ^*) for $\mu > 0$ sufficiently small, the condition

$$(\mu^{**}) \quad \int_{-\infty}^{\infty} [u_t^2(s,x) + u_x^2(s,x) + u_{tt}^2(s,x) + \cdots + u_{xxx}^2(s,x)]dx$$

$$+ \int_0^s \int_{-\infty}^{\infty} [u_t^2(t,x) + u_{tt}^2(t,x) + \cdots + u_{xxx}^2(t,x)]dxdt \leq \mu^2 \quad (0 \leq s < T)$$

is satisfied. The inequality (μ^{**}) in turn implies that condition (μ^*) holds and the cycle closes in a standard manner using Proposition 3.1. Thus the maximal interval of existence of the solution $u(t,x)$ is $[0,\infty) \times \mathbb{R}$ and (μ^{**}) holds for $0 \leq s < \infty$. This proves properties (i) and (ii) and Theorem 2.1.

To establish the a priori inequality (μ^{**}) we shall work with the equivalent form (3.2) of (HF) and we require some additional properties of the resolvent kernel k.

Lemma 3.3. *Let assumptions* (a) *be satisfied and let* k *be the resolvent kernel of* a' *defined by equation* (k). *Then*

(i) $k \in B^2[0,\infty)$, $k(0) = -\dfrac{a'(0)}{[a(0)]^2} > 0$,

(ii) $k(t) = k_\infty + K(t)$, $k_\infty = \dfrac{1}{\hat{a}(0)} > 0$; $K^{(m)} \in L^1(0,\infty)$, $m = 0,1,2$,

(iii) *for every* $T > 0$ *and for every* $v \in L^2[0,T]$ *there exists a number* $\gamma > 0$ *such that*

(3.8) $$\int_0^T v(t)\,\frac{d}{dt}\,(k*v)(t)dt \geq \gamma \int_0^T v^2(t)dt\ .$$

Remarks on Lemma 3.3. We refer to [6, Lemma 3.1] for the proof of Lemma 3.3 and to [1, Lemma 2.1] for some comments and corrections of that proof. Here we make some additional comments concerning the energy inequality (3.8) which is of independent interest. If, as is the case here $k' \in L^1(0,\infty)$, the inequality (3.8) is derived by the following simple argument (see the method of [10, Theorem 1]). Extend k' evenly for $t < 0$, and let

$$v_T(t) = \begin{cases} v(t) & \text{if } t \in [0,T] \\ 0 & \text{otherwise}\ . \end{cases}$$

Then

$$\int_0^T v(t)\,\frac{d}{dt}\,(k*v)(t)dt = k(0)\int_0^T v^2(t)dt + \int_0^T v(t)(k'*v)(t)dt$$

$$= k(0)\int_0^T v^2(t)dt + \frac{1}{2}\int_0^T v(t)\int_0^T k'(t-\tau)v(\tau)d\tau dt$$

$$= k(0)\int_{-\infty}^{\infty} v_T^2(t)dt + \frac{1}{2}\int_{-\infty}^{\infty} v_T(t)\int_{-\infty}^{\infty} k'(t-\tau)v_T(\tau)d\tau dt\ .$$

Letting $\tilde{v}_T(\eta) = \int_{-\infty}^{\infty} e^{-i\eta t}v_T(t)dt$, $(\eta \in \mathbb{R})$, the Parseval and convolution theorems give

$$\int_0^T v(t)\,\frac{d}{dt}\,(k*v)(t)dt = \frac{k(0)}{2\pi}\int_{-\infty}^{\infty} |\tilde{v}_T(\eta)|^2 d\eta + \frac{1}{4\pi}\int_{-\infty}^{\infty} |\tilde{v}_T(\eta)|^2\tilde{k}'(\eta)d\eta\ .$$

But $\tilde{k}'(\eta) = 2\mathrm{Re}\,\widehat{k'(i\eta)}$, where $\hat{}$ is the Laplace transform, and $\mathrm{Re}\,\widehat{k'}(i\eta) = \mathrm{Re}[i\eta\hat{k}(i\eta) - k(0)]$. Therefore,

$$\int_0^T v(t)\,\frac{d}{dt}\,(k*v)(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\tilde{v}_T(\eta)|^2\mathrm{Re}[i\eta\hat{k}(i\eta)]d\eta\ .$$

Now an easy calculation from equation (k) yields

$$\mathrm{Re}\ i\eta\hat{k}(i\eta) = \mathrm{Re}\,\frac{1}{\hat{a}(i\eta)} = \frac{\mathrm{Re}\ \hat{a}(i\eta)}{|\hat{a}(i\eta)|^2}\ .$$

Thus, to prove the ineqaulity (3.8) it suffices to establish the existence of $\gamma > 0$ such that

$$\frac{\mathrm{Re}\ \hat{a}(i\eta)}{|\hat{a}(i\eta)|^2} \geq \gamma > 0 \qquad (\eta \in \mathbb{R}) .$$

This is precisely what is done with the aid of assumptions (a) in [6, Lemma 3.1], although Mac Camy's derivation of (3.8) is different from the above and unnecessarily complicated. The above suggests that the frequency domain condition (2.1) in assumptions (a) should be replaced by the condition

(S) there exists $\alpha > 0$ such that $\alpha\ \mathrm{Re}\ \hat{a}(i\eta) \geq |\hat{a}(i\eta)|^2 \quad (\eta \in \mathbb{R})$.

The importance of condition (S) was first recognized by O. J. Staffans [11] in a different context. He showed [11, Theorem 2] that condition (S) is satisfied for at least two classes of kernels of importance for the problem (HF)

(i) $\begin{cases} a \in L^1(0,\infty) \cap BV[0,\infty) \text{ and } a \text{ \underline{strongly positive} on } [0,\infty), \\ \text{i.e. there exists an } \varepsilon > 0 \text{ such that } \mathrm{Re}\ \hat{a}(i\eta) \geq \varepsilon(1+\eta^2)^{-1}, \end{cases}$

and

(ii) $\begin{cases} a \in L^1(0,\infty) \text{ and } a \text{ and } -a' \text{ are nonnegative and convex} \\ \text{on } (0,\infty) \text{ (here } a(0+) = -a'(0+) = +\infty \text{ are allowed)} . \end{cases}$

Staffans also gives an example of a kernel which is a positive definite measure μ satisfying (S), but such that μ is <u>not</u> strictly positive $(\mathrm{Re}\ \hat{\mu}(i\eta) > 0)$.

Incidentally, it is not hard to show that if $a \in L^1(0,\infty)$, $a(0) > 0$, and a is either strongly positive on $[0,\infty)$ or a satisfies condition (S), then $a'(0) < 0$. It is also important to notice that if a satisfies (S) and $\hat{a}(i\eta)$ is defined as a function (e.g. if $a \in L^1(0,\infty)$), then $\hat{a}(i\eta)$ can vanish at most on a set of measure zero on the imaginary axis.

The above considerations suggest that the energy inequality (3.8) is true under other useful conditions which are much more general than assumptions (a), and such results are now being obtained.

The remainder of this section is devoted to the derivation of the a priori estimates which imply (μ^{**}). Define

$$W(w) = \int_0^w \sigma(\xi)d\xi \geq \frac{p_0}{2} w^2 \qquad w \in [-c_0, c_0] . \tag{3.9}$$

where the inequality follows from (3.7). Let u be a local solution of (3.2) satisfying (μ^*) for some $T > 0$ and $0 < \mu < c_0$. Multiply (3.2) by u_t and integrate over $[0,s] \times \mathbb{R}$. Using (3.9) and Lemma 3.3 (iii) one obtains the estimate (recall we are doing the special case $f \equiv 0$ in (HF) so that $u_t(0,x) \equiv 0$)

$$(3.10) \qquad \frac{1}{2}\int_{-\infty}^{\infty} u_t^2(s,x)dx + a(0)\int_{-\infty}^{\infty} W(u_x(s,x))dx + \gamma\int_0^s\int_{-\infty}^{\infty} u_t^2(t,x)dxdt$$

$$\leq a(0)\int_{-\infty}^{\infty} W(u_x(0,x))dx \qquad (0 \leq s < T) ,$$

where an integration by parts with respect to x was carried out in the term

$$\int_0^s\int_{-\infty}^{\infty} \sigma(u_x(t,x))_x u_t(t,x)dxdt ,$$

followed by an application of (3.9). It follows from (3.9), (3.10) that

$$(3.11) \qquad \int_{-\infty}^{\infty} u_t^2(s,x)dx + a(0)p_0\int_{-\infty}^{\infty} u_x^2(s,x)dx + 2\gamma\int_0^s\int_{-\infty}^{\infty} u_t^2(t,x)dxdt$$

$$\leq 2a(0)\int_{-\infty}^{\infty} W(u_{0x}(x))dx \qquad (0 \leq s < T) .$$

Thus, for as long as (μ^*) holds with $\mu < c_0$, the quantities

$$\int_{-\infty}^{\infty} u_t^2(s,x)dx, \int_{-\infty}^{\infty} u_x^2(s,x)dx, \int_0^s\int_{-\infty}^{\infty} u_t^2(t,x)dxdt$$

are controllably small, uniformly on $[0,T)$.

We now derive two additional estimates, the first by differentiating (3.2) with respect to t, multiplying by $u_{tt}(t,x)$ and integrating over $[0,s] \times \mathbb{R}$, the second by differentiating (3.2) with respect to x, multiplying by $u_{tx}(t,x)$ and integrating over $[0,s] \times \mathbb{R}$. Following the procedure in obtaining (3.10), and noting that since $u_t(0,x) \equiv 0$ one now has

$$\frac{\partial^2}{\partial t^2}(k*u_t)(t,x) = \frac{\partial}{\partial t}(k*u_{tt})(t,x) ,$$

we obtain

$$(3.12)\qquad \frac{1}{2}\int_{-\infty}^{\infty} u_{tt}^2(s,x)dx + \frac{a(0)}{2}\int_{-\infty}^{\infty} \sigma'(u_x(s,x))u_{tx}^2(s,x)dx$$

$$+ a(0)\gamma\int_0^s\int_{-\infty}^{\infty} u_{tt}^2(t,x)dxdt \le \frac{1}{2}\int_{-\infty}^{\infty} u_{tt}^2(0,x)dx$$

$$+ \frac{a(0)}{2}\int_0^s\int_{-\infty}^{\infty} \sigma''(u_x(t,x))u_{tx}^3(t,x)dxdt \qquad (0 \le s < T)\ ,$$

and

$$(3.13)\qquad \frac{1}{2}\int_{-\infty}^{\infty} u_{tx}^2(s,x)dx + \frac{a(0)}{2}\int_{-\infty}^{\infty} \sigma'(u_x(s,x)u_{xx}^2(s,x)dx$$

$$+ a(0)\gamma\int_0^s\int_{-\infty}^{\infty} u_{tx}^2(t,x)dxdt \le \frac{a(0)}{2}\int_{-\infty}^{\infty} \sigma'(u_{0x}(x))u_{0xx}^2(x)dx$$

$$+ \frac{a(0)}{2}\int_0^s\int_{-\infty}^{\infty} \sigma''(u_x(t,x))u_{tx}(t,x)u_{xx}^2(t,x)dxdt \qquad (0 \le s < T)\ .$$

We add up (3.12), (3.13) and we claim that in the resulting inequality, and as long as (μ^*) holds with μ sufficiently small, each term on the right-hand side is either controllably small or can be majorized by the sum of such a quantity and a quantity that is dominated by one of the dissipation terms. Indeed, since from (3.3)

$$(3.14)\qquad u_{tt}(0,x) = a(0)\sigma(u_{0x})_x\ ,$$

the $L^2(\mathbb{R})$ norm of $u_{tt}(0,x)$ is controllably small. The two space-time integrals in (3.12), (3.13) are majorized as follows

$$(3.15)\qquad \left|\frac{1}{2}\int_0^s\int_{-\infty}^{\infty} \sigma''(u_x(t,x))u_{tx}^3(t,x)dxdt\right|$$

$$\le \frac{\mu}{2}\max_{[-c_0,c_0]}|\sigma''(\cdot)|\int_0^s\int_{-\infty}^{\infty} u_{tx}^2(t,x)dxdt\ ,$$

$$(3.16)\qquad \left|\frac{1}{2}\int_0^s\int_{-\infty}^{\infty} \sigma''(u_x(t,x))u_{tx}(t,x)u_{xx}^2(t,x)dxdt\right|$$

$$\le \frac{\mu}{2}\max_{[c_0,c_0]}|\sigma''(\cdot)|\int_0^s\int_{-\infty}^{\infty} u_{xx}^2(t,x)dxdt\ .$$

To estimate the integral on the right-hand side of (3.16) we have from (3.3)

$$(3.17)\quad a(0)\sigma'(u_x(t,x))u_{xx}(t,x) = u_{tt}(t,x) + a(0)k(0)u_t(t,x) + a(0)(k'*u_t)(t,x)\ ,$$

and this yields from (3.7) and standard estimates

$$(3.18)\quad a^2(0)p_0^2 \int_0^s \int_{-\infty}^{\infty} u_{xx}^2(t,x)dxdt \le 4 \int_0^s \int_{-\infty}^{\infty} u_{tt}^2(t,x)dxdt$$

$$+ 4a^2(0)k^2(0) \int_0^s \int_{-\infty}^{\infty} u_t^2(t,x)dxdt + 4a^2(0)\left(\int_0^{\infty} |k'(t)|dt\right)^2 \int_0^s \int_{-\infty}^{\infty} u_t^2(t,x)dxdt\ .$$

The restrictions imposed on μ are expressed in terms of parameters fixed a priori. For example, the estimate (3.15) and the desire to absorb this term into the corresponding dissipation term in (3.13), impose the restriction $\mu \max\limits_{[-c_0,c_0]} |\sigma''(\cdot)| \le \frac{\gamma a(0)}{4}$. The combination of (3.12)-(3.18) then yields that the quantities $\int_{-\infty}^{\infty} u_{tt}^2(s,x)dx$, $\int_{-\infty}^{\infty} u_{tx}^2(s,x)dx$, $\int_{-\infty}^{\infty} u_{xx}^2(s,x)dx$, $\int_0^s \int_{-\infty}^{\infty} u_{tt}^2(t,x)dxdt$, $\int_0^s \int_{-\infty}^{\infty} u_{tx}^2(t,x)dxdt$, and $\int_0^s \int_{-\infty}^{\infty} u_{xx}^2(t,x)dxdt$ are controllably small uniformly on $[0,T)$, provided (μ^*) holds for μ sufficiently small.

To get the final set of estimates one assumes temporarily that the additional smoothness assumption (3.5) of Proposition 3.2 is satisfied, so that u satisfies (3.6). We form the second derivative of equation (3.2) first with respect t, multiply by $u_{ttt}(t,x)$ and then integrate over $[0,s] \times \mathbb{R}$, secondly with respect to t and x, multiply by $u_{ttx}(t,x)$, and then integrate over $[0,s] \times \mathbb{R}$. Following the above procedure and using equation (3.3) to compute $u_{ttt}(0,x)$ and $u_{ttx}(0,x)$ a tedious but straightforward calculation (see [1], section 4) shows that the quantities $\int_{-\infty}^{\infty} u_{ttt}^2(s,x)dx$, $\int_{-\infty}^{\infty} u_{ttx}^2(s,x)dx$, $\int_{-\infty}^{\infty} u_{txx}^2(s,x)dx$, $\int_{-\infty}^{\infty} u_{xxx}^2(s,x)dx$, $\int_0^s \int_{-\infty}^{\infty} u_{ttt}^2 dxdt$, $\int_0^s \int_{-\infty}^{\infty} u_{ttx}^2 dxdt$, $\int_0^s \int_{-\infty}^{\infty} u_{txx}^2 dxdt$, $\int_0^s \int_{-\infty}^{\infty} u_{xxx}^2 dxdt$ are controllably small uniformly on $[0,T)$, provided (μ^*) holds for μ sufficiently small. Moreover, the detailed estimates show they depend solely on parameters which do not involve the additional assumption (3.5). Therefore, a simple density argument can be used to remove the extraneous assumption (3.5).

Combining all the controllable estimates obtained above yields the inequality (μ^{**}), for any local solution u satisfying (μ^{*}) for $\mu > 0$ sufficiently small. This completes the sketch of the proof of Theorem 2.1.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A mathematical model for nonlinear heat flow in a rigid body of material with memory leads to the integrodifferential equation problem → next page

$$\text{(HF)} \begin{cases} u_t(t,x) = \int_0^t a(t-\tau)\sigma(u_x(t,x))_x d\tau + f(t,x) & (0 < t < \infty,\ x \in \mathbb{R}) \\ u(0,x) = u_0(x) & (x \in \mathbb{R}), \end{cases}$$

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20. ABSTRACT - Cont'd.

which is analyzed by an energy method developed jointly with C. M. Dafermos. Global existence, uniqueness, boundedness and the decay of smooth solutions as $t \to \infty$ are established for sufficiently smooth and "small" data, under physically reasonable assumptions.

approaches infinity